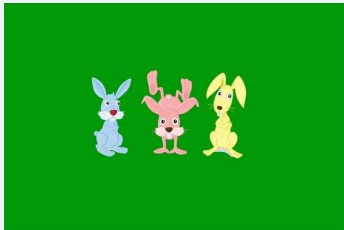


OMM - Populacioni modeli (jedna vrsta)

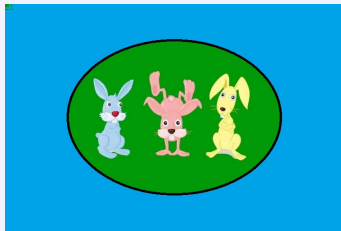
March 20, 2023



Malthusov model
(eksponencijalni)

$$\frac{dN(t)}{dt} = pN(t), N(0) = N_0$$

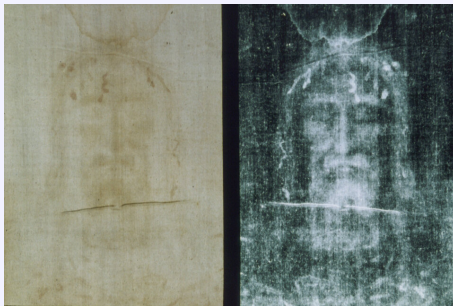
$$N(t) = N(0) \cdot e^{pt}$$



Verhulstov model
(logistički)

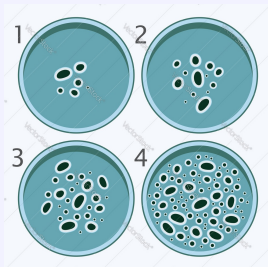
$$\frac{dN(t)}{dt} = rN(t) \left(1 - \frac{N(t)}{K} \right), N(0) = N_0$$

$$N(t) = \frac{K}{1 + \left(\frac{K}{N(0)} - 1 \right) \cdot e^{-rt}}$$



Izračunati starost Torinskog pokrova ako u uzorku količina atoma C14 predstavlja 92% količine atoma C12.

Vreme poluraspada C14 iznosi 5730 godina.

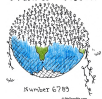


$t(\text{min})$	$N(t)$
0	1.99
16	2.68
32	3.63
48	4.89
64	6.63
80	8.93
96	12.10

Posmatramo kulturu bakterija i izmerenu količinu bakterija za date vremenske trenutke.

Ako je poznato da se bakterije umnožavaju prateći eksponencijalni model, odrediti $N(t)$ (uz uslov da im se okruženje ne menja).

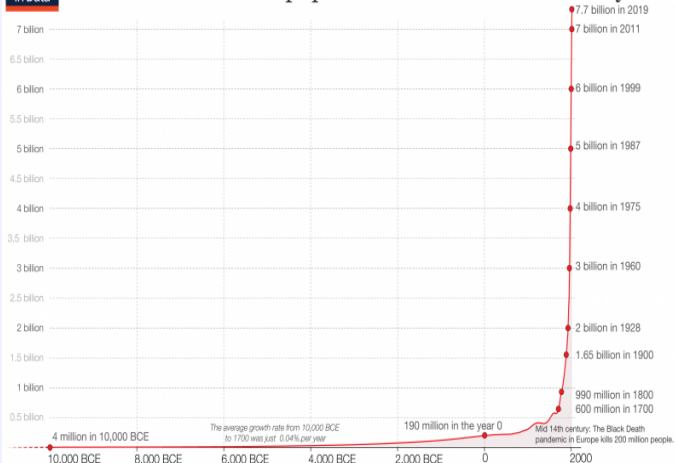
OVERPOPULATION



Ima li mesta/resursa na zemlji za sve?

Our World
in Data

The size of the world population over the last 12,000 years



Based on estimates by the History Database of the Global Environment (HYDE) and the United Nations. On OurWorldInData.org you can download the annual data.

This is a visualization from OurWorldInData.org, where you find data and research on how the world is changing.

Licensed under CC-BY-SA by the author Max Roser.



Kameno doba



Poljoprivredna revolucija (cc pre 10.000 god)



Kuga (14. vek)



Industrijska revolucija (cc 1700)



Zelena revolucija (cc 1950)

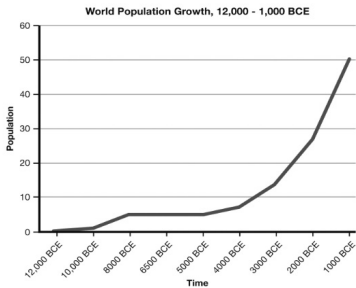
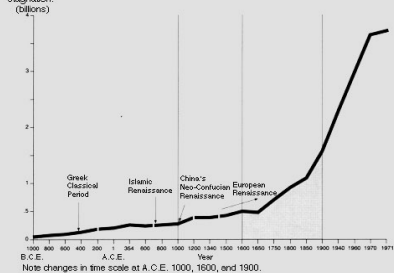
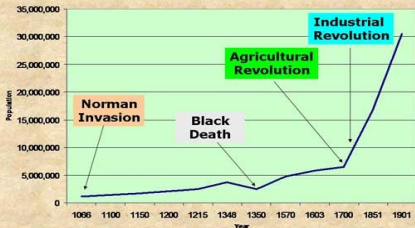


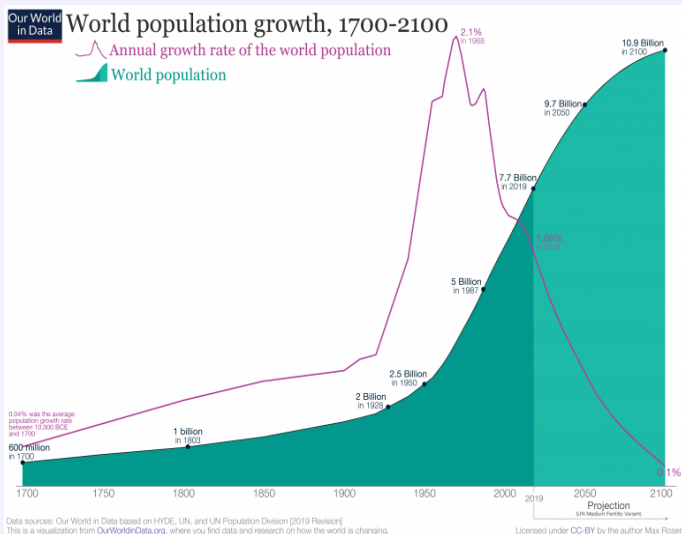
FIGURE 2. Actual world population growth, showing Renaissance impulses and periods of stagnation.



Estimated Population of England 1066 to 1900

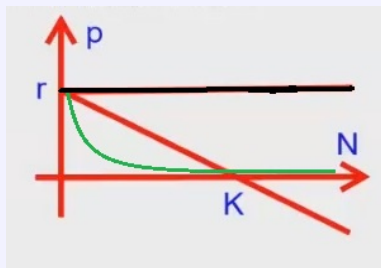


Šta nas čeka?



Gompertz model (1825)

- Osiguranja, rast biljaka, rast tumora, ...



- eksponencijalni
- logistički
- Gompertz

$$\frac{dN}{dt} = p(t)N(t), \quad N(0) = N_0$$

$$\frac{dp}{dt} = -\alpha \cdot p(t), \quad p(0) = p_0$$

 \Rightarrow

$$N(t) = N_0 e^{\frac{p_0}{\alpha}(1 - e^{-\alpha t})}$$

Rast tumora

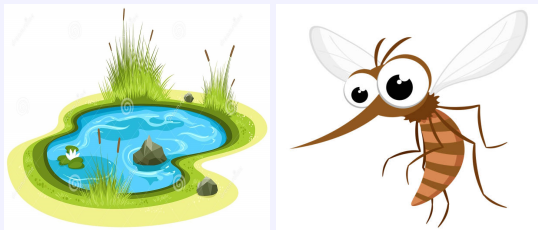
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```

Koji model koristiti?

Kako utiče okruženje?



- Jezero/baro je stanište za razmnožavanje komaraca.
- Komarci se razmnožavaju prateći logistički model.
- Jezero/baro se smanjuje/povećava u zavisnosti od godišnjeg doba.

Kako utiče okruženje? (cont.)

Kako se menja broj komaraca?

$$\frac{dN}{dt} = rN(t) \left(1 - \frac{N(t)}{K(t)} \right), \quad N(0) = N_0$$

Kako se menja jezero/bara (stanište)?

- $\frac{dK}{dt} = 0$, $K(0) = K$ - ne menja se u vremenu, $K = const$
- $\frac{dK}{dt} = a$, $K(0) > 0$, $a = const$ - menja se konstantno
- $\frac{dK}{dt} = aK$, $a > 0$ - menja se eksponencijalno
- $\frac{dK}{dt} = a \cdot \cos(bt + c)$, $a, b, c = const$ - menja se periodično (godišnja doba)
- ...

Žetva (Harvest)



U jezeru živi vrsta ribe sa godišnjim prirastajem $r = 0.09$. Maksimalni kapacitet jezera je $K = 350$ riba. Svake godine se ulovi $H = 2$ jedinke.

Ako u trenutku t_0 ima $N_0 = 100$ riba, koliko će biti riba u trenutku $T = 10$ godina?

U kom trenutku t će broj riba u jezeru biti $N(t) = 200$?

Kako odrediti maksimalni broj jediniki koji sme da se ulovi a da ne dođe do istrebljenja vrste?

Polje pravaca?